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A Fuzzy Chip Controller for Nonlinear Vibrations

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Abstract. The fuzzy logic ability to manage nonlinear vibration control problems has one major drawback: the slow reaction time offered by a software-implemented controller. This justifies the adoption of integrated fuzzy chip controllers. Nevertheless, designing a fuzzy controller for structural control purposes requires some fine tuning work. This can be conveniently carried out over an equivalent electronic circuit which emulates a civil structure, rather than by an expensive laboratory environment for testing frame specimens or by using computer simulations which would not allow a real-time study. Conceiving and implementing an electronic circuit equivalent to a single-degree-of-freedom system is the topic of this paper.

A case study is finally presented. It discusses the major features of the adopted 'fuzzy chip controller'. A brief overview of the results of the laboratory tests is also given. They were performed in order to properly tune the fuzzy project with civil structure applications.

Keywords: Fuzzy chip, fuzzy controller, fuzzy logic, nonlinear vibration, vibration control.

1. Introduction

Fuzzy logic theory [17, 18, 22, 25] has been widely proposed for the active control of structural systems [5, 7, 11]. As an alternative to the classical control theory, it allows the resolution of imprecise or uncertain information [1] and, for earthquake engineering applications, it can handle the hysteretic behaviour of structures under seismic excitation [10].

The main advantages in adopting a fuzzy control scheme in civil engineering problems [14–16] can be summarized as follows:

- the uncertainties of input data from the ground motion and structural vibration sensors [1] are treated in a much easier way by the fuzzy control theory than by the classical control theory. Fuzzy logic, which is the basis of fuzzy controllers, intrinsically accounts for such uncertainties. The implementation of fuzzy controllers is based on linguistic synthesis and therefore they are not affected by the choice of a specific mathematical model. As a consequence, the resulting fuzzy controller possesses inherent robustness;
- the time delay due to feedback acquisition and processing and the need for a response dead-band are intrinsically defined as system characteristics;
- the control action can be designed as a bounded function of the state variables; this provides an appropriate model for the actual behaviour of the actuators;
- the feedback can be restricted to a subset of the state variables; in this way, the problem of not being able to measure some of them (as it occurs, for example, for the auxiliary variable required by a hysteretic idealization) is overcome;

– a knowledge base identifies the actual variables driving the control process: in the specific problem developed throughout this paper, only two variables must be measured and estimated to implement the controller.

Indeed, there are no special difficulties in implementing the controller required by structural applications into a software tool [19], but the slow reaction time offered by this approach prevents one from making a laboratory test feasible. This suggested the idea of looking for an integrated fuzzy chip [3, 6] and its development environment, where the controller can be implemented in a hardware form.

Designing the controller is a trial and error process which requires many repeated tests, some of them resulting in the failure of the system. It is therefore evident that there is a need for a testing device different from the real structure or the laboratory specimen. Three main solutions were envisaged:

1. a numerical model for the structural system; it produces the response in digital form to be used directly as an input to the fuzzy board; the noise affecting the real case could also be modelled and digitally simulated in a stochastic sense;
2. a numerical model for the structural system, the sensors and the actuator which together form the control closed loop, thus achieving a numerical model of the physical problem which results in more accuracy; again, the response obtained by using the numerical model is fed as input to the fuzzy board;
3. the adoption of an electronic circuit that emulates the structural system; the response is produced in analog form and is used directly as input to the fuzzy board in tests which are performed in real time.

The processing of the numerical model requires a relatively large computation effort in the first case, and an even larger one in the second. It results in a long time being taken for any test to be carried out and, more importantly, it prevents real-time tests. The time-scale must be artificially enlarged and all the ensuing variations (time delays, rate dependency of the response, etc.) must be added to the model. By contrast, the development of an analog tool turns out not to be expensive and is easy to use, providing adequate flexibility and allowing complete system analysis. The development of the last approach is pursued in this paper. The use of a simple dedicated electronic board to emulate the mechanical system allows the fuzzy controller to be developed and experimentally evaluated without need for resorting to more complex and costly computer-based systems.

It should be pointed out that the present goal is to develop a system where the closed-loop deviation from the quiescent position, resulting from an external stimulus, is minimized. Therefore, the governing equations can be linearized, even in the presence of large excitations. However, as shown at the end of Section 3, some system nonlinearities can be explicitly taken into account. This will be useful in further steps of the research, when a sort of optimization of the control energy will be pursued and excursions in the nonlinear range will be allowed for this purpose.

2. Governing Equations

2.1. The Mechanical System

Let a physical system be given, together with the specifications of its desired behaviour. A nonlinear dynamic system is governed by a set of first-order differential equations:

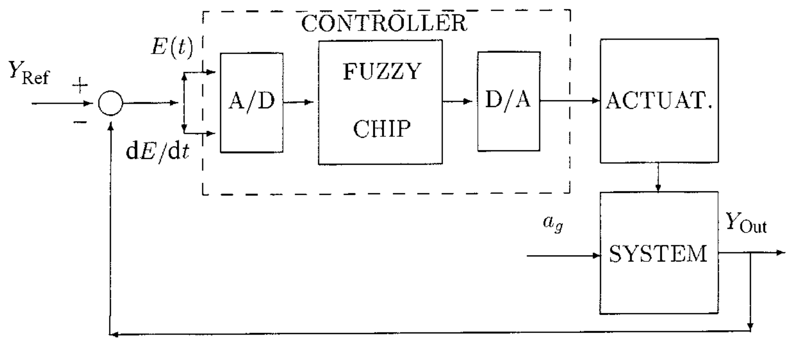

Figure 1. Closed-loop control system.

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t), \tag{1}$$

where the mathematical model $\mathbf{f}$ depends on the vector of the state variables, $\mathbf{x}$, the vector of the control action, $\mathbf{u}$, and the parameter t, the time. In the design of an active controller, the goal is to reduce the structural response amplitude, i.e. accelerations, velocities and displacements, at suitable points. This must be pursued under the limitation of both the number of measured signals and the control action level (limited by the actuator features and by the required amount of energy).

The long-term goal of the present research is to implement, in a laboratory environment, structural components which are controlled by devices driven by a fuzzy controller. In particular, the following schemes are pursued:

- The specimen frame in [4] (a three-storey frame with the possibility of adding braces at each level, to reduce the number of degrees of freedom of the system) is controlled by an active mass driver on the top. It is presently supported by an elastomeric bearing. When the braces are mounted, the whole system is modelled by a single-degree-of-freedom (SDOF) oscillator with hysteretic constitutive law (the bearing force-displacement relationship). A successful control would drive the structure to work in the initial linear range, thus allowing a linear idealization of the mechanical system.
- A pointer arm is located over a plane truss frame. Under external in-plane excitation, the frame response is controlled by air jets [21], but the arm connection point moves. Since the pointer must always be directed to the same fixed point, the resulting nonlinear response must be suitably controlled. Also in this case, an efficient control algorithm allows a linear idealization.

One specific objective of the present paper was therefore the design and implementation of an active control strategy for a nonlinear SDOF system:

$$\dot{x}_1(t) = x_2(t), \quad \dot{x}_2(t) = -h(x_1, x_2) + u(t) - a_g(t), \tag{2}$$

x_1 being the displacement, x_2 the velocity, their function $h(\,)$ a sort of restoring force, $u(t)$ the control term (an acceleration in the present form), and $a_g(t)$ the external absolute acceleration (i.e. the seismic ground acceleration).

The elementary feedback control scheme for such a system is depicted in Figure 1. Analog-to-digital (A/D) and digital-to-analog (D/A) converters are required as both input and output signals of the fuzzy processor chip used are digital, while real-world signals are analog.

The aim of the controller is to minimize the measure of the difference: $E(t) = Y_{\mathrm{Ref}}(t) - Y_{\mathrm{Out}}(t)$, Y denoting the measurable state variable, and the subscripts referring to its target and measured value, respectively. As one can see, one of the inputs to the controller is the variable $E(t)$. In order to take the dynamics of the system into account, a second input representing its time derivative is considered. The control feedback signals throughout this paper are displacements and velocities. In real implementations, the measurable quantities are often accelerations: an appropriate extension toward this goal was also conceived [8] and its implementation is presently in progress. Also, developing a fuzzy controller for the system in Equation (2) can be easily extended to multi-degree-of-freedom systems. This aspect is discussed in the Appendix.

2.2. Fuzzy Project

In the analysis of nonlinear control systems no general method is available for designing nonlinear controllers. One just has a collection of alternative and complementary techniques. For this purpose, expert control makes effective use of symbolic computation as a classic application of artificial intelligence in the design process of any control algorithm.

In controlling mechanical systems, the control strategy consists in the intuitive axiom that, if displacement and velocity are negative (with respect to a fixed Cartesian reference), the need is for a positive control force. If the input variables are zero, the control force should be zero, and so on. However, it is also equally intuitive that the intensity of the control force should be coherent with the degree of membership of the input variables to the negative or positive fuzzy subsets.

In particular, the fuzzy controller design was carried out within this research by assigning three fuzzy subsets with associated reference terms (NE = negative, ZE = zero, PO = positive) to the input variables (displacement and velocity). Five fuzzy subsets (PL = positive large and NL = negative large were added to the previous three) were associated to the output variable (the control force). After defining the variables and the related membership functions, the fuzzy rules determining the control can be written. Under the hypothesis of the presence of a single actuator, the inference rules can be defined in terms of a table where the columns correspond to the reference terms (or labels) associated with the first input variable (displacement), and the rows to the labels of the second input variable (velocity). The table entries are the corresponding reference terms for the output variable [5, 11].

Once the combination rule for fuzzy inference has been selected and the code has been loaded in fuzzy chip memories, the defuzzification step can be initiated accordingly.

3. Equivalent Circuit

The design of a fuzzy controller is pursued by emulating the behavior of the structural system by means of an equivalent electronic circuit. This approach allows the performance of the system under design to be evaluated and optimized with no need for expensive and time-consuming laboratory tests on (full-scale or scaled) real structures or for numerical simulations based on models of the structural system and the control components. These numerical simulations, in particular, would force one to stretch the time-scale [9] with the consequent request of accurate models for phenomena such as time delay, rate dependency and so on, models which do not always result in being accurate enough. The equivalent circuit should be as simple as possible to allow easy implementation and flexible use.

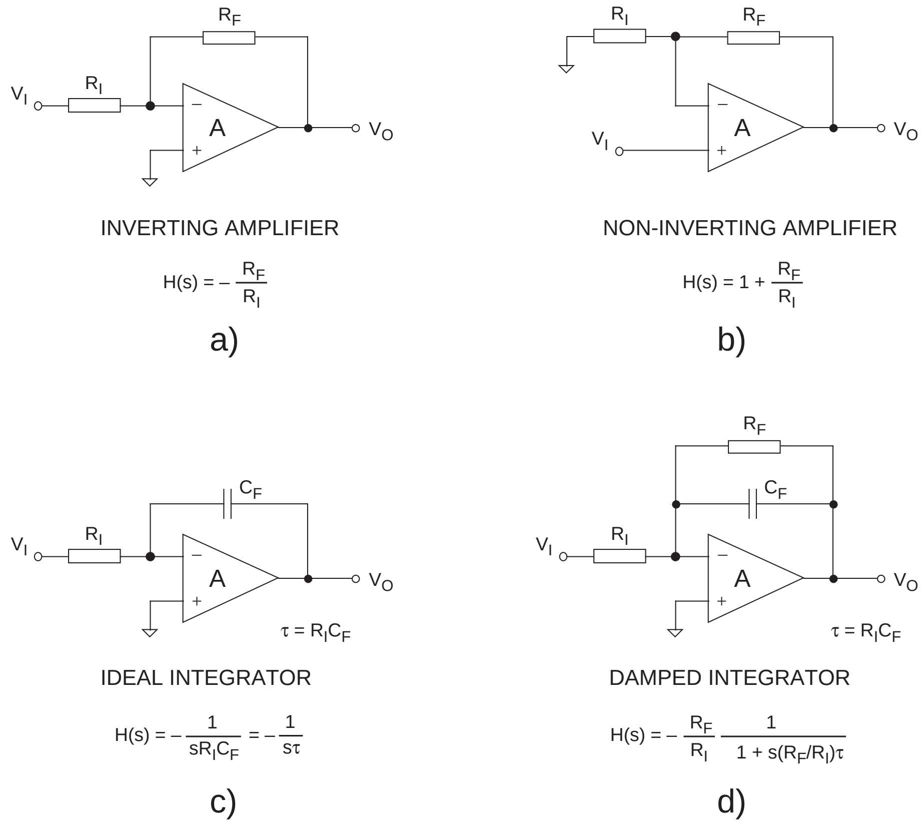

Figure 2. Basic active building blocks for electronic circuits: (a) inverting amplifier; (b) noninverting amplifier; (c) ideal integrator; (d) damped integrator.

An electronic circuit which emulates the original SDOF structural system was therefore conceived and implemented on a board. The starting point was the following system, which represents the linearization of Equation (2) around the origin:

$$\dot{x}_1(t) = x_2(t), \quad \dot{x}_2(t) = -ax_1(t) - bx_2(t) + u(t) - a_g(t), \tag{3}$$

where $a = k/M$ [1/s^2] and $b = c/M$ [1/s], k being the structural system stiffness, M its mass, and c its damping.

The equivalent circuit must drive the fuzzy controller, whose inputs are voltage signals. In electronic boards, they are conveniently generated using operational amplifiers (*op-amps*) connected in feedback configuration. The most popular configurations are the inverting amplifier (Figure 2a), the noninverting amplifier (Figure 2b), the ideal (or lossless) integrator (Figure 2c), and the damped (or lossy) integrator (Figure 2d) [23]. The corresponding transfer functions $H(s) = V_{\mathrm{O}}(s)/V_{\mathrm{I}}(s)$ in the Laplace domain, i.e. the ratios of output voltage over input voltage, are also given in Figure 2, where $\tau = R_I C_F$ (with the meaning of the symbols specified in Figure 2) is the integrator time constant. It is worth pointing out that, for $|sR_F C_F| = |s(R_F/R_I)\tau| \gg 1$, the transfer function of a damped integrator becomes substantially equal to that of an ideal integrator.

Equation (3) must be adequately manipulated to ensure easy implementation based on the above building blocks. If one now introduces the variable transformation [m] $\rightarrow$ [V] and, hence, $x_1(t) \rightarrow v_1(t)$, $x_2(t) \rightarrow v_2(t)/\tau$, $u(t) \rightarrow v_c(t)/\tau^2$ and $-a_g(t) \rightarrow v_{ag}(t)/\tau^2$ (where

$v_1(t)$, $v_2(t)$, $v_c(t)$ and $v_{ag}(t)$ represent voltages and τ is a suitable time constant, which is chosen as shown below), one obtains

$$\dot{v}_1(t) = \frac{v_2(t)}{\tau}, \quad \frac{\dot{v}_2(t)}{\tau} = -av_1(t) - b\frac{v_2(t)}{\tau} + \frac{v_c + v_{ag}}{\tau^2}. \tag{4}$$

Taking the Laplace transform of both equations and dividing all members by s (the current variable in the Laplace domain), one has

$$V_1 = \frac{V_2}{s\tau}, \quad \frac{V_2}{\tau} = -a\frac{V_1}{s} - b\frac{V_2}{s\tau} + \frac{V_c + V_{ag}}{s\tau^2}. \tag{5}$$

and, therefore,

$$V_1 = \frac{V_2}{s\tau}, \quad V_2 = -a\tau^2\frac{V_1}{s\tau} - b\tau\frac{V_2}{s\tau} + \frac{V_c + V_{ag}}{s\tau}, \tag{6}$$

where the factor $1/s\tau$ has been highlighted in all terms at the right side. Equation (6) can be rewritten as

$$V_1 = \frac{V_2}{s\tau}, \quad V_2 = -\alpha\frac{V_1}{s\tau} - \beta\frac{V_2}{s\tau} + \frac{V_c + V_{ag}}{s\tau}, \tag{7}$$

where $\alpha = a\tau^2$ and $\beta = b\tau$ are dimensionless positive constants. Equation (7a) can be directly implemented by an inverting ideal integrator (Figure 2c)) followed by a unity-gain inverting amplifier (Figure 2a)), thus providing the first required output signal, $v_1(t)$.

The second output, $v_2(t)$, can be obtained by solving Equation (7) for V_2, which gives

$$V_2 = -\frac{\alpha}{s\tau + \beta}V_1 + \frac{1}{s\tau + \beta}(V_c + V_{ag}) \tag{8}$$

or

$$\begin{aligned} V_2 &= -\frac{\alpha}{\beta}\frac{1}{1 + s(\tau/\beta)}V_1 + \frac{1}{\beta}\frac{1}{1 + s(\tau/\beta)}(V_c + V_{ag}) \\ &= -\frac{1}{\beta}\frac{1}{1 + s(\tau/\beta)}[\alpha V_1 - (V_c + V_{ag})]. \end{aligned} \tag{9}$$

This equation can be implemented by a damped integrator having two inputs (signals v_1 and $v_c + v_{ag}$ with adequate weight factors). To achieve a non-inverting relationship between $(v_c + v_{ag})$ and v_2, an additional unity-gain inverting amplifier is needed.

Figure 3 shows the electrical scheme of the complete equivalent circuit where the used component values ensure the exact implementation of Equations (7a) and (9) ($\tau = RC$). It is worth pointing out that the obtained circuit is intrinsically stable (provided that the used op-amps are stable), as the feedback loop contains an odd number of inversions.

The time constant τ determines the gain of the integrators at any given frequency. Its value was chosen as the best trade-off to ensure adequate gain to the system in the whole frequency range of interest (0.5 to 25 Hz). More specifically, the value of this constant was set as equal to $1/(2\pi \cdot f_0)$, where $f_0 = 3.53$ Hz is the geometric mean of the two bound frequencies in the specified range. A single integrated circuit (TL084, quad op-amp with JFET input stage) was used to implement all the operational amplifiers. The implemented equivalent circuit played a basic role in the design and optimization of the fuzzy controller. Thanks to its use, the membership definition and the fuzzy reasoning could be optimized following a ‘wait and

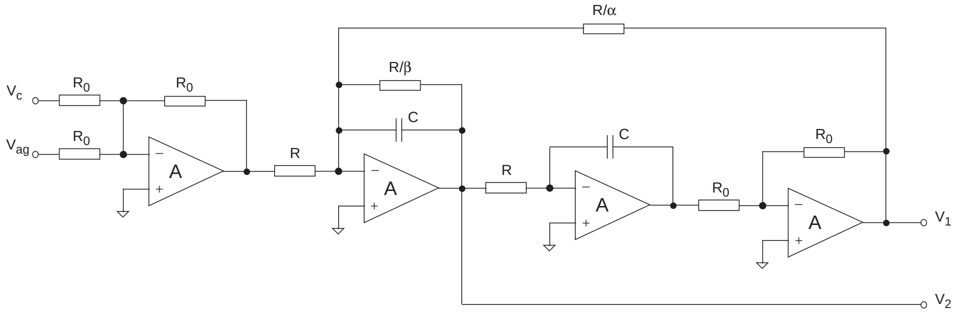

Figure 3. Electronic circuit emulating a structural SDOF system.

see' approach. Moreover, the effect of noise could be investigated as well as the adoption of techniques for reducing its effects.

When required, nonlinearities can be also taken into account by using nonlinear components. For example, piecewise linear waveshaping circuits [13, 23] can be used for this purpose. A piecewise linear voltage-to-current characteristic can be provided by a string of conventional (linear) resistors, each having a diode connected in parallel. When a modest voltage is applied across the resistor string, all diodes are turned off and, hence, do not substantially affect the string operation. When the voltage across a given resistor reaches the cut-in voltage V_γ of its parallel connected diode, this is turned on and behaves like a battery V_B ($V_B = V_\gamma$) with a small series resistance (the diode dynamic forward resistance), thereby decreasing the incremental resistance of the string. Hysteretic behavior can also be emulated by using positive-feedback circuits.

One of the referees of this paper was wondering about the actual possibility of extending the previous developments to multi-degrees-of-freedom systems. The referee suggested to the authors to add an appendix with these details, even if the paper itself is focused on an SDOF investigation.

4. Case Study

4.1. Fuzzy Chip

The fuzzy project for the control of the SDOF system and of its equivalent electronic circuit has been carried out with a commercially available fuzzy kit [12, 24]. It consists of a hardware component, the Fuzzy Programmable Board (FPB), and a software development environment. The FPB is a low-cost general-purpose board employing a dedicated digital 8-bit micro-controller which can be used both as a fuzzy coprocessor and as a stand-alone microcontroller. In the former case, it can work together with a standard microprocessor performing normal control tasks, and will be independently responsible for all the fuzzy related computing. The fuzzy processor core includes the fuzzyfier, the inference unit and the defuzzyfier. Running this device involves a downloading and an on-line phase. The downloading phase allows its programming in terms of I/O settings, universe of discourse, membership functions and rules. During this phase, the fuzzy processor prepares its internal memories for the on-line processing phase and loads its program memories with the microcode generated by the compiler. The project for the fuzzy coprocessor is generated by using a user-friendly environment composed of a complete set of software tools and an application development

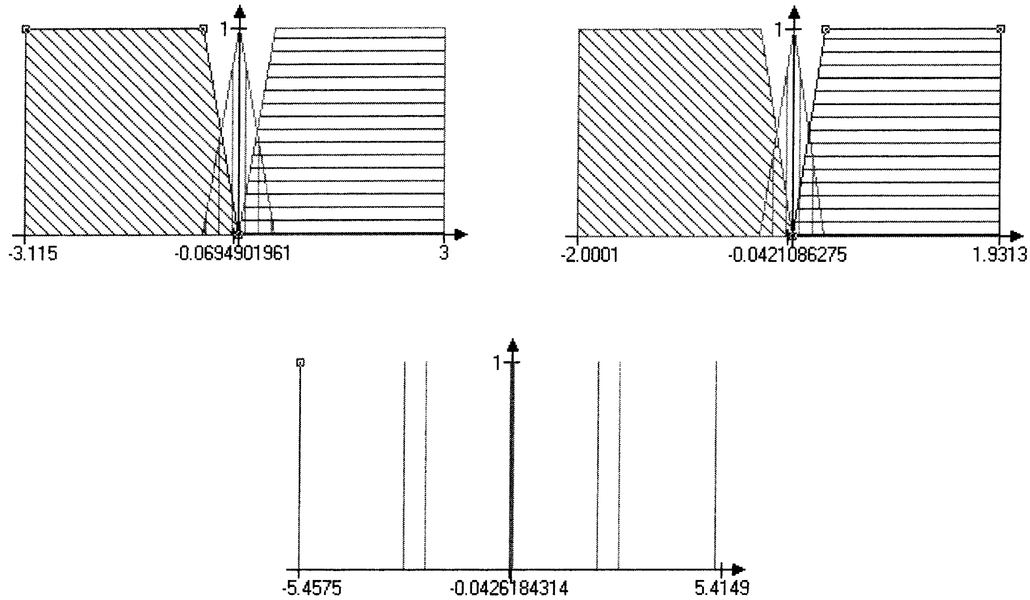

Figure 4. Top: membership functions for the input variables (displacement left; velocity right). Bottom: crisp values for the output variables.

board. It uses typical fuzzy logic terminology and objects, allowing the processor memories to be programmed coherently with the control specifications defined by the designer. The graphic output produced by the fuzzy project development tool for the control problem under investigation is depicted in Figure 4. As one can see, the used fuzzy processor allows the definition of triangular/trapezoidal membership functions for the input variables, while the output variables may only assume crisp values.

4.2. Tuning

A number of preliminary tests were carried out in order to evaluate the performance of the fuzzy controller and to tune the parameters of interest. In particular, in order to test the control action provided by the fuzzy chip, the following tests were conducted:

1. Fully numerical simulation. This test has been performed by exploiting the exporter facility provided by the fuzzy development system, which allows the codified fuzzy project to be exported to different program environments (Matlab, C, Fuzzy Logic Language FLL) in order to perform simulation and validation. In particular, the project was translated into a C++ source code and a complete closed-loop simulation was performed. In this numerical simulation, the differential SDOF system was solved by means of the Runge–Kutta integration method [20]. The results of this test validated the fuzzy project conceived for this special application [4].
2. Control of the numerical model by the fuzzy processor. The numerical model of the SDOF structural system was used, while the control loop was closed by the hardware fuzzy controller and the real data acquisition board. No A/D and D/A converters were used, as in this system all data are processed in the digital domain. This test allowed the hardware module's functionality to be evaluated.

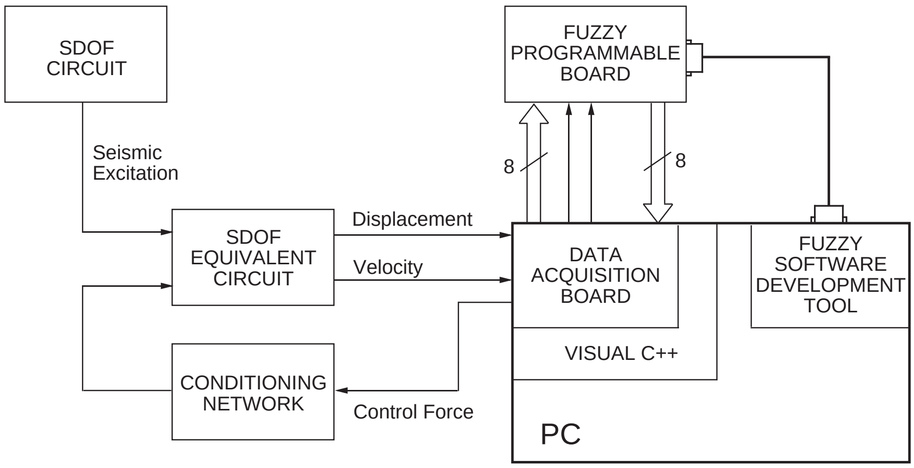

Figure 5. Control scheme incorporating the equivalent circuit.

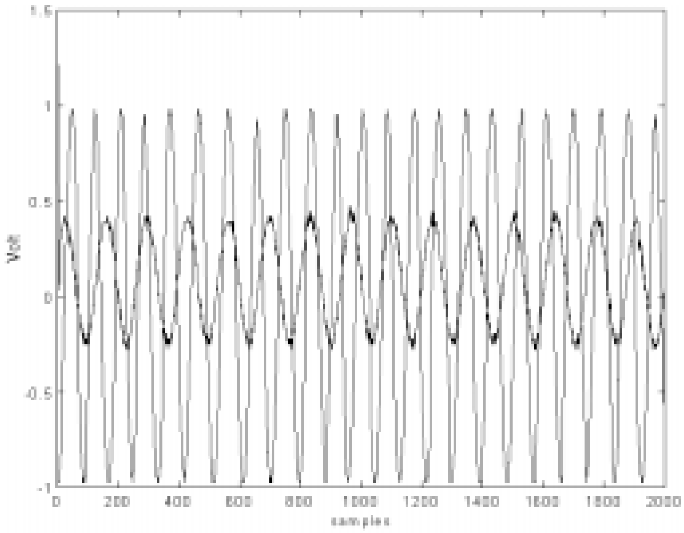

Figure 6. Controlled and uncontrolled (larger response) displacement.

3. Control of the equivalent electronic circuit by the fuzzy processor. The differential analytical model was eventually replaced by the equivalent electronic circuit [8]. The scheme is shown in Figure 5. The data acquisition board provides the required A/D conversion of the circuit output variables ($v_1(t), v_2(t)$), which are the fuzzy processor input, and the D/A conversion of the fuzzy processor output variable ($v_c(t)$), which is the control input of the equivalent circuit. It also ensures correct data transmission from/to the fuzzy processor through an 8-bit data bus. A conditioning circuit was used to shift the quiescent output voltage of the data acquisition board to the value required by the circuit equivalent to the mechanical SDOF system.

The input was defined as a simple sine wave for the tuning phase. Figure 6 shows the results of the control action in terms of mechanical displacement. The numerical data were mass $M = 40{,}000$ kg; stiffness $k = 2333$ kN/m; damping $c = 0.0093k$ kN s/m [2]; the frequency of the excitation was 3.53 Hz.

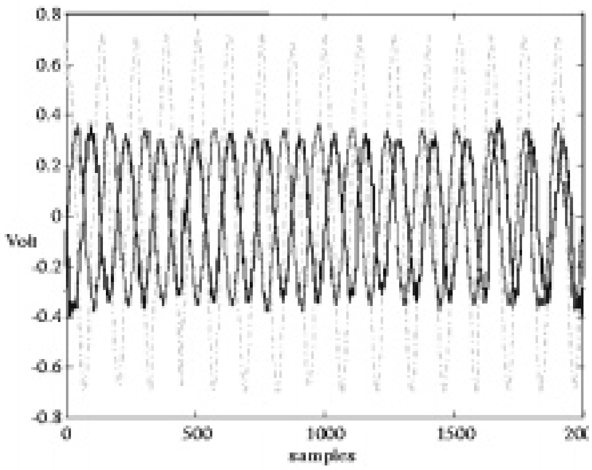

Figure 7. Effect of narrowing the central MF of the displacement variable (from the largest response (dotted line), to the smallest one).

In order to improve the control action performance, a number of adjustments to the fuzzy project were performed. Figure 7 shows the effect of narrowing the central membership function (MF) (see Figure 4) on the control action for the displacement variable. The narrower the MF becomes, the stronger the control action does. A further test was performed by increasing the number of MFs for the displacement variable. This only led to a slight improvement in the control action.

4.3. Broad Spectrum Testing

Once the fuzzy project was improved and validated, the simple input sine waveform was replaced by a broadband signal (frequency range from 0.5 to 25 Hz). As one can see in Figure 8, even in this case the control action on the displacement variable can be significantly appreciated.

5. Conclusions

An electronic circuit emulating the behavior of an SDOF mechanical system has been conceived and implemented. It is coupled with a fuzzy chip in order to tune the fuzzy project so as to control the voltage response in the best way. The fuzzy chip is then ready to control the vibration of the original mechanical system.

The results of a number of laboratory tests validating the effectiveness of the control scheme have been presented.

Improvements presently in progress cover multi-degrees-of-freedom systems and the possibility of using sensors which measure accelerations rather than displacements and velocities.

Appendix. The MDOF Case

The basic extension to a multi-degree-of-freedom (MDOF) system is pursued for a shear type idealization of a multistorey frame. The notation is complicated by adding a second index

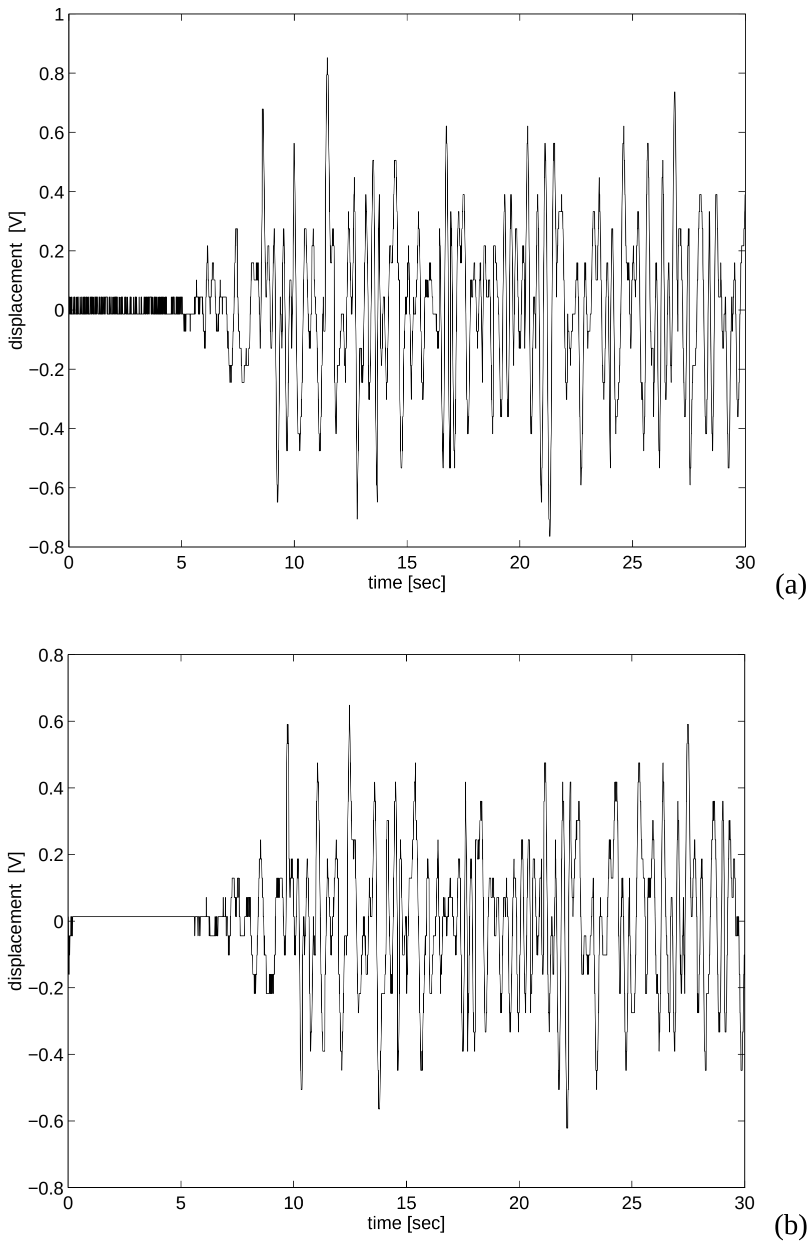

Figure 8. Uncontrolled (a) and controlled (b) displacement time histories under white-noise excitation.

ranging from 1 (the top storey, where the control force is applied) to 3 (the storey directed linked with the basement). Again, the basement is excited by the ground acceleration a_g.

$$\begin{aligned}
\dot{x}_{11}(t) &= x_{21}(t), \\
\dot{x}_{12}(t) &= x_{22}(t), \\
\dot{x}_{13}(t) &= x_{23}(t), \qquad (10)
\end{aligned}$$

$$\begin{aligned}
\dot{x}_{21}(t) &= -a_1(x_{11}(t)-x_{12}(t)) - b_1(x_{21}(t)-x_{22}(t)) + u(t) - a_g(t),\\
\dot{x}_{22}(t) &= -a_1(x_{12}(t)-x_{11}(t)) - a_2(x_{12}(t)-x_{13}(t)) - b_1(x_{22}(t)-x_{21}(t))\\
&\quad - b_2(x_{22}(t)-x_{23}(t)) - a_g(t),\\
\dot{x}_{23}(t) &= -a_2(x_{13}(t)-x_{12}(t)) - a_3x_{13}(t) - b_2(x_{23}(t)-x_{22}(t)) - b_3x_{23}(t) - a_g(t).
\end{aligned} \tag{11}$$

Introducing the variable transformation of Section 3, taking the Laplace transform of both sets of equations and dividing all members by s (the current variable in the Laplace domain), one has

$$\begin{aligned}
V_{11} &= \frac{V_{21}}{s\tau},\\
V_{12} &= \frac{V_{22}}{s\tau},\\
V_{13} &= \frac{V_{23}}{s\tau},\\
\frac{V_{21}}{\tau} &= -a_1\frac{V_{11}-V_{12}}{s} - b_1\frac{V_{21}-V_{22}}{s\tau} + \frac{V_c+V_{ag}}{s\tau^2},\\
\frac{V_{22}}{\tau} &= -a_1\frac{V_{12}-V_{11}}{s} - a_2\frac{V_{12}-V_{13}}{s} - b_1\frac{V_{22}-V_{21}}{s\tau} - b_2\frac{V_{22}-V_{23}}{s\tau} + \frac{V_{ag}}{s\tau^2},\\
\frac{V_{23}}{\tau} &= -a_2\frac{V_{13}-V_{12}}{s} - a_3\frac{V_{13}}{s} - b_2\frac{V_{23}-V_{22}}{s\tau} - b_3\frac{V_{23}}{s\tau} + \frac{V_{ag}}{s\tau^2}.
\end{aligned} \tag{12}$$

The second set of three similar equations becomes

$$\begin{aligned}
V_{21} &= -\alpha_1\frac{V_{11}-V_{12}}{s\tau} - \beta_1\frac{V_{21}-V_{22}}{s\tau} + \frac{V_c+V_{ag}}{s\tau},\\
V_{22} &= -\alpha_1\frac{V_{12}-V_{11}}{s\tau} - \alpha_2\frac{V_{12}-V_{13}}{s\tau} - \beta_1\frac{V_{22}-V_{21}}{s\tau} - \beta_2\frac{V_{22}-V_{23}}{s\tau} + \frac{V_{ag}}{s\tau},\\
V_{23} &= -\alpha_2\frac{V_{13}-V_{12}}{s\tau} - \alpha_3\frac{V_{13}}{s\tau} - \beta_2\frac{V_{23}-V_{22}}{s\tau} - \beta_3\frac{V_{23}}{s\tau} + \frac{V_{ag}}{s\tau},
\end{aligned} \tag{13}$$

where $\alpha_i = a_i\tau^2$ and $\beta_i = b_i\tau$ are dimensionless positive constants. The previous equation can be rewritten as

$$\begin{aligned}
V_{21} &= -\frac{\alpha_1}{s\tau+\beta_1}V_{11} + \frac{\alpha_1}{s\tau+\beta_1}V_{12} + \frac{\beta_1}{s\tau+\beta_1}V_{22} + \frac{1}{s\tau+\beta_1}(V_c+V_{ag}),\\
V_{22} &= -\frac{\alpha_1+\alpha_2}{s\tau+\beta_1+\beta_2}V_{12} + \frac{\alpha_1}{s\tau+\beta_1+\beta_2}V_{11} + \frac{\alpha_2}{s\tau+\beta_1+\beta_2}V_{13}\\
&\quad + \frac{\beta_1}{s\tau+\beta_1+\beta_2}V_{21} + \frac{\beta_2}{s\tau+\beta_1+\beta_2}V_{23} + \frac{1}{s\tau+\beta_1+\beta_2}(V_{ag}),\\
V_{23} &= -\frac{\alpha_2+\alpha_3}{s\tau+\beta_2+\beta_3}V_{13} + \frac{\alpha_2}{s\tau+\beta_2+\beta_3}V_{12}\\
&\quad + \frac{\beta_2}{s\tau+\beta_2+\beta_3}V_{22} + \frac{1}{s\tau+\beta_2+\beta_3}(V_{ag})
\end{aligned} \tag{14}$$

or

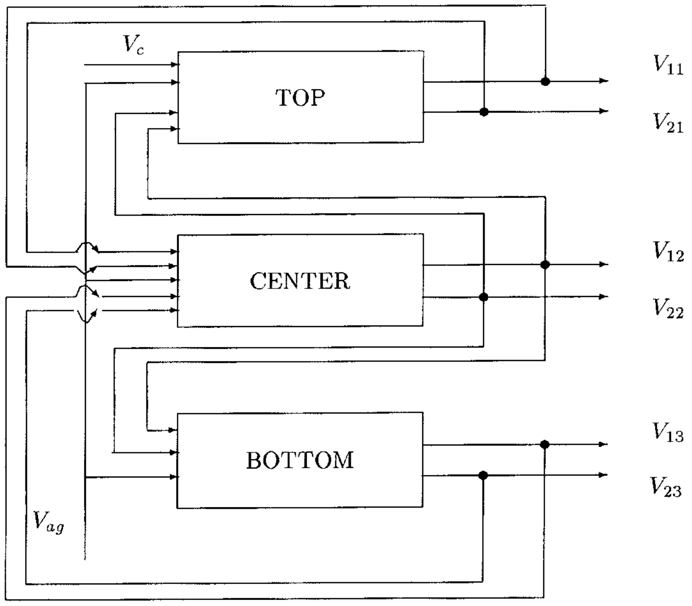

Figure 9. Scheme for the implementation of an electronic circuit equivalent to a three-degree-of-freedom mechanical system.

$$\begin{aligned}
V_{21} &= -\frac{1}{\beta_1}\frac{1}{1+s(\tau/\beta_1)}[\alpha_1 V_{11} - \alpha_1 V_{12} - \beta_1 V_{22} - (V_c + V_{ag})],\\
V_{22} &= -\frac{1}{\beta_1+\beta_2}\frac{1}{1+s[\tau/(\beta_1+\beta_2)]}\\
&\times\ [(\alpha_1+\alpha_2)V_{12} - \alpha_1 V_{11} - \alpha_2 V_{13} - \beta_1 V_{21} - \beta_2 V_{23} - V_{ag}],\\
V_{23} &= -\frac{1}{\beta_2+\beta_3}\frac{1}{1+s[\tau/(\beta_2+\beta_3)]}[(\alpha_2+\alpha_3)V_{13} - \alpha_2 V_{12} - \beta_2 V_{22} - V_{ag}],
\end{aligned} \tag{15}$$

which can be directly implemented in an electronic circuit (Figure 9).

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